A Separation Logic to Verify Termination of Busy-Waiting for Abrupt Program Exit

Tobias Reinhard
KU Leuven
imc-DistriNet Research Group
tobias.reinhard@kuleuven.be

Amin Timany
Aarhus University
Logic and Semantics Group
timany@cs.au.dk

Bart Jacobs
KU Leuven
imc-DistriNet Research Group
bart.jacobs@kuleuven.be

Abstract
Programs for multiprocessor machines commonly perform busy-waiting for synchronisation. In this paper, we make a first step towards proving termination of such programs. We approximate (i) arbitrary waitable events by abrupt program termination and (ii) busy-waiting for events by busy-waiting to be abruptly terminated.

We propose a separation logic for modularly verifying termination (under fair scheduling) of programs where some threads eventually abruptly terminate the program, and others busy-wait for this to happen.

1 Introduction
Programs for multiprocessor machines commonly perform busy-waiting for synchronisation [15, 16, 18]. In this paper, we make a first step towards proving termination of such programs. Specifically, we propose a separation logic [17, 20] for modularly verifying termination (under fair scheduling) of programs where some threads eventually abruptly terminate the program, and others busy-wait for this to happen.

Here, by modular we mean that we reason about each thread and each function in isolation. That is, we do not reason about thread scheduling or interleavings. We only consider these issues when proving the soundness of our logic.

In this work, we approximate (i) arbitrary events that a program might wait for by abrupt termination and (ii) busy-waiting for events by busy-waiting to be abruptly terminated. In Section 6, we sketch preliminary ideas for generalizing this to verifying termination of busy-waiting for arbitrary events, and how this work may also be directly relevant to verifying liveness properties of a program’s I/O behaviour.

Throughout this paper we use a very simple programming language to illustrate our verification approach. Its simplicity would allow us to verify termination of busy-waiting for abrupt termination via a static analysis significantly simpler than the proposed separation logic. However, in contrast to such an analysis, our approach is also applicable to realistic languages. We are confident that the logic we propose can be combined straightforwardly with existing concurrent separation logics like Iris [9] to verify termination of realistic programs where threads busy-wait for abrupt program termination.

2 The Language
We consider a simple programming language with an exit command that abruptly terminates all running threads, a fork command, a looping construct loop skip to express infinite busy-waiting loops and sequencing c1; c2.

Definition 2.1 (Commands and Continuations). We denote the sets of commands c and continuations κ as defined by the grammar presented in Figure 1 by Cmnds and Κ. We consider sequencing · as defined in the grammars of commands and continuations to be right-associative.

We use commands and continuations to represent programs and single threads, respectively, as well as natural numbers for thread IDs. Continuation done marks the end of a thread’s execution. We consider thread pools to be functions mapping a finite set of thread IDs to continuations.

Definition 2.2 (Thread Pools). We define the set of thread pools TP as follows:

\[ TP := \{ P : \Theta \rightarrow K \mid \Theta, c_{\text{fin}} \subseteq \mathbb{N} \} \]

We denote thread pools by P, thread IDs by θ and the empty thread pool by θ_{tp} : ∅ → K.

Definition 2.3 (Thread Pool Extension). Let \( P : \Theta \rightarrow K \in TP \) be a thread pool. We define:

- \( P_{tp} \emptyset := P \)
- \( P_{tp}(\{ \kappa \}) : \Theta \cup \{ \max(\Theta) + 1 \} \rightarrow K \) with \( (P_{tp}(\{ \kappa \}))(\theta) = P(\theta) \) for all \( \theta \in \Theta \) and \( (P_{tp}(\{ \kappa \}))(\max(\Theta) + 1) = \kappa \).

Figure 1. Syntax

\[ c \in Cmnds := \text{exit} \mid \text{loop } \text{skip} \mid \text{fork } c \mid c ; c \]
\[ \kappa \in K := \text{done } \mid c ; \kappa \]
we need to assume fair scheduling.

The code snippet spawns a new thread which will abruptly sequence of thread pools such that \( P \) terminates if the exiting thread is eventually scheduled. Hence, are scheduled. Meanwhile, the presented program only terminate the entire program and then busy-waits for the program to be abruptly terminated. The operational semantics defined in Figure 2. A reduction step has the form

\[
\text{RedST-Seq} \quad (c_1; c_2) \mapsto c_1; (c_2; \kappa), \emptyset
\]

Figure 2. Reduction rules for single threads.

\[
\begin{align*}
\text{RedST-Lift} & \quad \theta \in \text{dom}(P) \quad P(\theta) = \kappa \quad \kappa \mapsto_{st} \kappa', T \\
& \quad P \circ_{tp} \theta \mapsto_{tp} P[\theta := \kappa'] +_{tp} T \\
\text{RedST-Exit} & \quad \theta \in \text{dom}(P) \quad P(\theta) = \text{exit;} \kappa \\
& \quad P \circ_{tp} \theta \mapsto_{tp} \emptyset
\end{align*}
\]

Figure 3. Reduction rules for thread pools.

- \( P \circ_{tp} \theta' : \Theta \setminus \{\theta'\} \rightarrow K \) with
  \( (P \circ_{tp} \theta')(\theta) = P(\theta) \)

We consider a standard small-step operational semantics for our language defined in terms of two reduction relations: (i) \( \mapsto_{st} \) for single thread reduction steps and (ii) \( \mapsto_{tp} \) for thread pool reduction steps.

**Definition 2.4** (Single-Thread Reduction Relation). We define a single-thread reduction relation \( \mapsto_{st} \) according to the rules presented in Figure 2. A reduction step has the form

\[
\kappa \mapsto_{st} \kappa', T
\]

for a set of forked threads \( T \subset K \) with \( |T| \leq 1 \).

**Definition 2.5** (Thread Pool Reduction Relation). We define a thread pool reduction relation \( \mapsto_{tp} \) according to the rules presented in Figure 3. A reduction step has the form

\[
P \circ_{tp} \theta \mapsto_{tp} P'
\]

for a thread ID \( \theta \in \text{dom}(P) \).

Figure 4 illustrates the type of programs we aim to verify. The code snippet spawns a new thread which will abruptly terminate the entire program and then busy-waits for the program to be terminated. The operational semantics defined above is non-deterministic in regard to when and if threads are scheduled. Meanwhile, the presented program only terminates if the exiting thread is eventually scheduled. Hence, we need to assume fair scheduling.

**Definition 2.6** (Reduction Sequence). Let \((P_i)_{i \in \mathbb{N}}\) be a sequence of thread pools such that \( P_i \circ_{tp} P_{i+1} \) holds for all

\[
\text{fork exit; loop skip}
\]

Figure 4. Example program with two threads: An exiting thread and one waiting for the program to be abruptly terminated.

\[i \in \mathbb{N} \\text{and some sequence } (\theta_i)_{i \in \mathbb{N}} \text{ of thread IDs. Then we call } (P_i)_{i \in \mathbb{N}} \text{ a reduction sequence.}\]

Note that according to this definition, reduction sequences are implicitly infinite.

**Definition 2.7** (Fairness). We call a reduction sequence \((P_i)_{i \in \mathbb{N}}\) fair iff for all \( k \in \mathbb{N} \) and \( \theta \in \text{dom}(P_k) \) there exists \( j \geq k \) such that

\[
P_j \circ_{tp} P_{j+1}.
\]

3 The Logic

In this paper, we develop a separation logic to reason about termination of busy-waiting programs. Separation logic is designed for reasoning about program resources as well as ghost resources [17, 20]. The latter is information attached to program executions for the purpose of program verification, e.g., a resource tracking how many threads have access to a shared memory location [8]. Here, we use ghost resources to track which thread will eventually exit, i.e., abruptly terminate the entire program.

**Obligations & Credits.** Remember that exit terminates all running threads. Therefore, in order to modularly reason about program termination we need information about other threads performing exit.

For this purpose, we introduce two kinds of ghost resources: obligations and credits. Threads holding an obligation are required to perform exit while threads holding a credit are allowed to busy-wait for another thread to exit. As seen in the next section we ensure that no thread (directly or indirectly) waits for itself.

We aggregate obligations into obligations chunks, where each obligations chunk collects the held obligations of a single thread.

**Assertions.** The language of assertions defined in the following allows us to express knowledge and assumptions about held obligations and credits. The language contains the standard separating conjunction \( \cdot \ast \cdot \) as well as two non-standard predicates obs and credit to express the possession of ghost resources. (i) \( \text{obs}(n) \) expresses the possession of one obligations chunk containing \( n \) exit obligations; i.e., it expresses that the current thread holds \( n \) exit obligations. (ii) \( \text{credit} \) expresses the possession of an exit credit that can be used to busy-wait for another thread to exit.

\(^1\)As outlined in Section 6, we plan to extend this logic to one where threads are obliged to set ghost signals. This makes it necessary to track how many signals remain to be set. Hence, we track the number of obligations.
In this section we present the proof system we propose for verifying termination of programs with busy-waiting for abrupt program exit and illustrate its application. Further, we present a soundness theorem stating that every program, which provably discharges all its exit obligations and starts without credits, terminates.

### 4 Verifying Termination of Busy-Waiting

A Separation Logic to Verify Termination of Busy-Waiting for Abrupt Program Exit

Figure 5. Syntax of assertions.

\[ a \in A := \text{True} \mid \text{False} \mid a \ast a \mid \text{obs}(n) \mid \text{credit} \]

\[ n \in \mathbb{N} \]

\[ R \models_{A} \text{True} \]

\[ R \models_{A} a \ast a \iff \exists R_1, R_2 \in R, R = R_1 \cup R_2 \land R_1 \models_{A} a_1 \land R_2 \models_{A} a_2 \]

\[ (O, \chi) \models_{A} \text{obs}(o) \iff o \in O \]

\[ (O, \chi) \models_{A} \text{credit} \iff \chi \geq 1 \]

Figure 6. Modeling relation for assertions.

**Definition 3.1 (Assertions).** Figure 5 defines the set of assertions \( A \).

As we see in Section 4 it is crucial to our verification approach that the obs-predicate captures a full obligations-chunk and that this chunk can only be split when obligations are passed to a newly forked thread. We represent the information about the held obligations chunks and credits by resource bundles \((O, \chi)\).

**Definition 3.2 (Resource Bundles).** We define the set of resource bundles \( R \) as

\[ R := \text{Bags}(\mathbb{N}) \times \mathbb{N}. \]

For \((O_1, \chi_1), (O_2, \chi_2) \in R\) we define

\[ (O_1, \chi_1) \models_{R} (O_2, \chi_2) := (O_1 \cup O_2, \chi_1 + \chi_2). \]

Threads hold exactly one obligations-chunk, i.e., resources \((O, \chi)\) with \(|O| = 1\). We call such resource bundles complete.

**Definition 3.3 (Complete Resource Bundles).** We call a resource bundle \((O, \chi) \in R\) complete if \(|O| = 1\) holds and write complete\((O, \chi)\).

Note that the following definition indeed ensures that the obs-predicate captures a full obligations-chunk. Hence, no bundle with one obligations-chunk can satisfy an assertion of the form \(\text{obs}(n) \ast \text{obs}(n')\).

**Definition 3.4 (Assertion Model Relation).** Figure 6 defines the assertion model relation \( \models_{A} \subseteq R \times A \). We write

\[ R \models_{A} a \]

to express that resource bundle \( R \in R \) models assertion \( a \in A \).

**Figure 7. View shift rules.**

**Hoare Triples.** We use Hoare triples \((A) c \{B\}\) [5] to specify the behaviour of programs. Such a triple expresses that given precondition \(A\), command \(c\) can be reduced without getting stuck and if this reduction terminates, then postcondition \(B\) holds afterwards. In particular, a triple \((A) c \{\text{False}\}\) expresses that \(c\) does not terminate normally, i.e., it either diverges or exits abruptly.

**Ghost Steps.** When verifying the termination of a program \(c\), we consider it to start without any obligations or credits, i.e., \((\text{obs}(0)) c \{B\}\). Obligation-credit pairs can, however, be generated during so-called ghost steps. These are steps that exclusively exist on the verification level and only affect ghost resources, but not the program’s behaviour [2,9]. A credit can also be cancelled against an obligation.

**View Shift.** In our proofs, we need to capture ghost steps as well as drawing conclusions from assertions, e.g., rewriting \(A \ast B\) into \(B \ast A\) and concluding \(\text{obs}(0)\) from the assumption False. We ensure this by introducing a view shift relation \(\Rightarrow\) [9]. A view shift \(A \Rightarrow B\) expresses that whenever \(A\) holds, then either (i) \(B\) also holds or (ii) \(B\) can be established by performing ghost steps. \(A \Leftrightarrow B\) stands for \(A \Rightarrow B \land B \Rightarrow A\).

**Definition 4.1 (View Shift).** We define the view shift relation \(\Rightarrow \subseteq A \times A\) according to the rules presented in Figure 7.

Note that view shifts only allow to spawn or remove obligations and credits simultaneously. This way, we ensure that the number of obligations and credits in the system remains equal at any time (provided this also holds for the program’s initial state).

**Proof Rules.** We verify program specifications \((A) c \{B\}\) via a proof relation \(\vdash\) defined by a set of proof rules. These rules are designed to prove that every command \(c\), which provably discharges its obligations, i.e., \(\vdash \{\text{obs}(n)\} c \{\text{obs}(0)\}\), terminates under fair scheduling.

**Definition 4.2 (Proof Relation).** We define a proof relation \(\vdash\) for Hoare triples \((A) c \{B\}\) according to the rules presented in Figure 8.

Obligation-credit pairs can be generated and removed via a ghost step by applying PR-ViewShift plus VS-ObCred. The only way to discharge an obligation, i.e., removing it
without simultaneously removing a credit, is via rule PR-Exit. That is, a discharging program \( \vdash \{ \text{obs}(1) \} c \{ \text{obs}(0) \} \), must involve an abrupt exit at some point.

We can pass obligations and credits to newly forked threads by applying PR-Fork. However, note that in order to prove anything about a command fork c, we need to prove that the forked thread discharges or cancels all of its obligations.

The only way to justify busy-waiting is via PR-Loop, which requires the possession of a credit. Note that the rule forbids the looping thread to hold any obligations. This ensures that threads do not busy-wait for themselves to exit. In other words, we can only prove a triple of the form \( \{A\} \text{ loop skip } \{B\} \) if precondition A expresses the knowledge that another thread is obliged to exit.

**Example.** Consider the program \( c_{ext} = \text{fork exit; loop skip} \) presented in Figure 4. It forks a new thread instructed to exit and busy-waits for it to do so. We can verify its termination under fair scheduling by proving \( \vdash \{ \text{obs}(0) \} c_{ext} \{ \text{obs}(0) \} \). Figure 9 sketches this proof. Note that the assumption of fair scheduling is essential, since otherwise we would have no guarantees that the exiting thread is ever executed.

The following soundness theorem states that we can prove termination of a program c under fair scheduling by proving that it discharging all its exit obligations, i.e., \( \vdash \{ \text{obs}(n) \} c \{ \text{obs}(0) \} \). By such a proof we verify that no fair infinite reduction sequence of c exists. That is, the reduction eventually terminates, either abruptly via exit or normally.

**Theorem 4.3** (Soundness). Let \( \vdash \{ \text{obs}(n) \} c \{ \text{obs}(0) \} \). There exists no fair reduction sequence \( (P_i)_{i \in \mathbb{N}} \) starting with \( P_0 = ((\theta_0, c; \text{done})) \) for any \( \theta_0 \in \mathbb{N} \).

Figure 9. Verification sketch for a program with two threads: An exiting thread and one busy-waiting for abrupt termination. Applied proof rules are highlighted in violet.

**5 Soundness**

In this section, we sketch the proof of Soundness Theorem 4.3 as well as all necessary concepts and lemmas. Formal definitions and proofs can be found in the technical report [19].

**Bridging the Gap.** According to our proof rules, ghost resources are not static but affected by the commands occurring in a program. For instance, forking allows us to pass resources to the newly forked thread and exiting discharges obligations. Hence, we annotate threads by resource bundles to obtain annotated thread pools and define annotated versions \( \sim_{\text{ast}} \) and \( \sim_{\text{alp}} \) of the relations \( \sim_{\text{st}} \) and \( \sim_{\text{tp}} \). We define \( \sim_{\text{alp}} \) as the union of two reduction relations: (i) \( \sim_{\text{real}} \) for actual program execution steps and (ii) \( \sim_{\text{ghost}} \) for ghost steps. These relations reflect the interaction between commands and ghost resources as reflected in the proof rules as well as ghost steps realized by view shifts.

Figure 10 shows three of the rules we use to define \( \sim_{\text{ast}} \), \( \sim_{\text{ghost}} \), and \( \sim_{\text{real}} \). A ghost step performed by GS-OnCredIntro spawns an obligation-credit pair, comparable to a verification ghost step realized by a view shift. Reduction rule A-RedSTLoop requires the looping thread to hold a credit but forbids it to hold any obligation. This ensures that busy-waiting threads do not wait for themselves and corresponds to the restriction that proof rule PR-Loop imposes on looping threads. Furthermore, we only allow annotated threads to terminate, i.e., be removed from the thread pool, if they do not hold any obligations, as shown by rule A-RedTP-THREADTerm. Note that in contrast to the plain semantics, reductions in the annotated semantics can get stuck.

The annotated semantics act as an intermediary between high-level reasoning steps, e.g., using obligations to track which thread is going to exit, and the actual program execution.

**Interpreting Hoare Triples.** We interpret Hoare triples in terms of a model relation \( \equiv_1 \) and an auxiliary safety relation safe(\( R, c \)). Intuitively, a continuation \( \kappa \) is safe under a
A Separation Logic to Verify Termination of Busy-Waiting for Abrupt Program Exit

We write with a resource bundle.

A Separation Logic to Verify Termination of Busy-Waiting for Abrupt Program Exit

By induction on the derivation of $\vdash H$. Proof.

call an annotated reduction sequence $(\text{forbids})$

Our definition of fairness for annotated reduction sequences $(\text{forbids})$

Soundness of Hoare Triples $(\text{forbids})$

rules also holds in our model. $(\text{forbids})$

$\vdash (\{a\}, \chi)$. $(\text{forbids})$

resources such that the reduction of $(R, \kappa)$ does not get stuck. $(\text{forbids})$

We write anno$(P^\kappa, P)$ to express that $P^\kappa$ is an annotated version of $P$, containing the same threads but each equipped with a resource bundle.

Definition 5.1 (Safety). We define the safety predicate $\text{safety} \subseteq R \times K$ coinductively as the greatest solution (with respect to $\subseteq$) of the following equation:

$$\text{safety}(R, \kappa) = \text{complete}(R) \rightarrow$$

$$\forall P', \forall \theta \in \text{dom}(P^\kappa), \forall P^\kappa :$$

$$P(\theta) = \kappa \land P \xrightarrow{\theta \text{ stp}} P' \land \text{anno}(P^\kappa, P) \land P^\kappa(\theta) = (R, \kappa) \rightarrow$$

$$\exists P_G^\kappa. \exists P^\kappa. P^\kappa \xrightarrow{\theta \text{ real}} P_G^\kappa \land P^\kappa \xrightarrow{\theta \text{ real}} P^\kappa' \land \text{anno}(P^\kappa', P') \land \forall (R', \kappa') \in \text{range}(P^\kappa) \land \text{safe}(R', \kappa')$$

Definition 5.2 (Hoare Triple Model Relation). We define the Hoare triple model relation $\vdash_H$ such that

$\vdash H \{ A \} c \{ B \}$

$\forall R_F. \forall \kappa. (\forall R_B. R_B \vdash A \rightarrow \text{safety}(R_B \cup R_F, \kappa))$

$\rightarrow (\forall R_A. R_A \vdash A \rightarrow \text{safety}(R_A \cup R_F, c; \kappa))$

Every specification $\{ A \} c \{ B \}$ we can derive with our proof rules also holds in our model.

Lemma 5.3 (Soundness of Hoare Triples). Let $\vdash \{ A \} c \{ B \}$. Then $\vdash H \{ A \} c \{ B \}$ holds.

Proof. By induction on the derivation of $\vdash \{ A \} c \{ B \}$. □

Constructing Annotated Executions. Given a fair reduction sequence $(P_i)_{i \in \mathbb{N}}$ starting with $P_0 = \{(\theta, c; \text{done})\}$, we can construct a fair annotated reduction sequence $(P^\kappa_i)_{i \in \mathbb{N}}$. Our definition of fairness for annotated reduction sequences forbids $(P^\kappa_i)_{i \in \mathbb{N}}$ to perform ghost steps forever.

Definition 5.4 (Fair Annotated Reduction Sequences). We call an annotated reduction sequence $(P^\kappa_i)_{i \in \mathbb{N}}$ fair iff for all

$k \in \mathbb{N}$ and $\theta \in \text{dom}(P^\kappa_i)$ there exists $j \geq k$ such that

$P^\kappa_j \xrightarrow{\theta} P^\kappa_{j+1}$. $(\text{forbids})$

Lemma 5.5. Let $\vdash_H \{ A \} c \{ B \}, R_A \vdash A$ and complete$(R_A)$. Furthermore, let $(P_i)_{i \in \mathbb{N}}$ be fair with $P_0 = \{(\theta, c; \text{done})\}$. There exists a fair annotated reduction sequence $(P^\kappa_i)_{i \in \mathbb{N}}$ with $P^\kappa_0 = \{(\theta, (R_A, c; \text{done}))\}$.

As next step, we show that a fair and infinite annotated reduction sequence such as $(P^\kappa_i)_{i \in \mathbb{N}}$ constructed above, must start with an initial credit. We do this by analysing the sequence’s program order graph $G((P^\kappa_i)_{i \in \mathbb{N}})$ sketched below.

Program Order Graph. In the program order graph $G((P^\kappa_i)_{i \in \mathbb{N}})$ of $(P^\kappa_i)_{i \in \mathbb{N}}$, every node $i$ represents the $i$th reduction step of the sequence, i.e., $P^\kappa_0 \xrightarrow{\theta_i} P^\kappa_{i-1}$. An edge has the form $(i, \theta, n, j)$ and expresses that one of the following hold:

- $P^\kappa_i \xrightarrow{\theta_j} P^\kappa_{i+1}$ is the first step of a thread forked in step $i$ or
- $j$ is the next index representing a reduction of thread $\theta_i$ (in which case $\theta_i = \theta_j$ holds).

In both cases, $n$ represents the name of the reduction rule applied in step $P^\kappa_i \xrightarrow{\theta_j} P^\kappa_{i+1}$. We set $0$ as the graph’s root.

Lemma 5.6. Let $G_P$ be a finite sibling-closed 2 prefix of some program order graph $G((P^\kappa_i)_{i \in \mathbb{N}})$ with $P^\kappa_0 = \{(\theta_0, (R_0, \kappa_0))\}$ for some complete $R_0$. For all $i \in \text{leaves}(G_P)$ choose $\theta_i, \alpha_i, \chi_i, \kappa_i$ such that $P^\kappa_i \xrightarrow{\theta_i} P^\kappa_{i+1}$ and $P^\kappa_i(\theta_i) = ((\{a\}, \chi), \kappa_i)$

The sum of the obligations held by the threads reduced in the leaves of $G_P$ equal the sum of the credits held by these threads:

$\sum_{i \in \text{leaves}(G_P)} \alpha_i = \sum_{i \in \text{leaves}(G_P)} \chi_i$

Proof. Proof by induction on the size of $G_P$. □

Lemma 5.7. There are no fair annotated reduction sequences $(P^\kappa_i)_{i \in \mathbb{N}}$ with $P^\kappa_0 = \{(\theta_0, ((\{a\}), 0, \kappa))\}$ for any $\theta_0, \alpha_0, \kappa$.

Proof Sketch. We prove our claim by contradiction. Suppose there exists such a reduction sequence $(P^\kappa_i)_{i \in \mathbb{N}}$. Consider the maximal loop-edge-free sibling-closed prefix $G^\triangledown$ of its program order graph. Since the $G^\triangledown$ has no loop edges, it is finite.

Since the reduction sequence $(P^\kappa_i)_{i \in \mathbb{N}}$ is infinite, at least one of the leaves of the prefix $G^\triangledown$ is the predecessor of a loop edge. This leaf (i.e., the thread reduced in the leaf) must therefore hold a credit but no obligations. By Lemma 5.6, for any finite prefix of $G$, the sum of obligations held by the leaves equals the sum of credits held by them. Hence, some other leaf $\alpha_o$ must hold an obligation. Every leaf is either (i) the final step of a thread or (ii) the predecessor of

$^2$A subgraph is sibling-closed if, for each node $n$ of the subgraph, $n$’s predecessors’ successors are also in the subgraph. In other words, for each fork step node, either both the next step of the forkling thread and the first step of the forked thread are in the subgraph, or neither are.


6 Future Work

We are currently formalizing the presented approach and its soundness proof in Coq.

**Ghost Signals.** We plan to extend the verification approach described in this paper to a verification technique we call *ghost signals*, which allows us to verify termination of busy-waiting for arbitrary events. Ghost signals come with an obligation to set the signal and a credit that can be used to busy-wait for the signal to be set. Consider a program with two threads: $t_e$ eventually performs some event $X$ (such as setting a flag in shared memory) and $t_w$ busy-waits for $X$. By letting $t_e$ set the signal when it performs $X$, and thereby linking the ghost signal to the event, we can justify termination of $t_w$’s busy-waiting.

**I/O Liveness as Abrupt Program Exit.** In concurrent work we encode I/O liveness properties as abrupt program termination following a conjecture of Jacobs et al. [7]. Consider a non-terminating server $S$ which shall reply to all requests. We can prove liveness of $S$ using the following methodology:

- For some arbitrary, but fixed $N$, assume that responding to the $N^{th}$ request abruptly terminates the whole program.
- Prove that the program always terminates.

One can combine this approach with the one of the present paper to verify liveness of a server where multiple threads independently receive and handle requests. Using a *prophecy variable* [10], one can determine ahead of time which thread will receive the exiting request. The other threads can then be seen as busy-waiting for this thread to exit.

**Combination.** We plan to combine the two approaches sketched above and conjecture that the combination will be expressive enough to verify liveness of programs such as the server $S$ presented in Figure 11. It runs a set of receiving and a set of responding threads communicating via a shared queue. The responding threads busy-wait for requests to arrive in the queue. In order to verify liveness of $S$, we need to show that some thread eventually abruptly terminates the program by acquiring and responding to the $N^{th}$ request. This requires us to prove that the responding threads’ busy-waiting for requests to arrive in the shared queue terminates.

In order to demonstrate the approach’s usability, we plan to implement it in VeriFast [6] and prove liveness of $S$.

7 Related Work

Liang and Feng [13, 14] propose LiLi, a separation logic to verify liveness of blocking constructs implemented via busy-waiting. In contrast to our verification approach, theirs is based on the idea of contextual refinement. LiLi does not support forking nor structured parallel composition.

D’Osualdo et al. [1] propose TaDA Live, a separation logic for verifying termination of busy-waiting. This logic allows to modularly reason about fine-grained concurrent programs and blocking operations that are implemented in terms of busy-waiting and non-blocking primitives. It uses the concept of obligations to express thread-local liveness invariants, e.g., that a thread eventually releases an acquired lock. TaDA Live is expressive enough to verify CLH and spin locks. The current version supports structured parallel composition instead of unstructured forking. Comparing their proof rules to ours, it is fair to say that our logic is simpler. Of course, theirs is much more powerful. We hope to be able to extend ours as sketched above while remaining simpler.

8 Conclusion

In this paper we proposed a separation logic to verify the termination of programs where some threads abruptly terminate the program and others busy-wait for abrupt termination. We proved our logic sound and illustrated its application.

Abrupt termination can be understood as an approximation of a general event. We outlined our vision on how to extend our approach to verify the termination of busy-waiting for arbitrary events. We have good hope that the final logic will be as expressive as TaDA Live proposed by D’Osualdo et al. [1] while remaining conceptually simpler.

Further, we sketched our vision to combine this extended work with concurrent work on the encoding of I/O liveness properties as abrupt termination. We illustrated that this combination will be expressive enough to verify liveness of concurrent programs where multiple threads share I/O responsibilities.
References


