

This book introduces, with some variations, eight mathematically flavoured games or puzzles. As the authors accurately explain in their preface, the type of problems they present look at first sight almost impossible to solve. It is only after a careful analysis, reducing it to a formal (say mathematical) reformulation, that it becomes clear that some solution strategy can be designed that is in some sense even optimal. So, when discussing the solution to a problem, it is taken as an excellent pretext to explain some piece of mathematics. To a reader with a minimal mathematical background, it is explained what a Hamming code is, what a cyclic group is, but also some elements are introduced from linear algebra, probability and even broader elements such as information theory, projective geometry, and algebraic topology. Thus, what starts as a playful game with a seemingly impossible solution becomes, after placing it in an appropriate mathematical context, relatively easy to solve. Moreover, by isolation and abstraction of the essentials, it is simple to consider more general situations. A better advertisement for the power of mathematics and a stronger motivation to study mathematical formalism and mathematical structures can hardly be found.

The book opens with a classic game, which is also delivering the title of the book. So let's take this as an illustration of the concept used by the authors throughout the book. Consider 3 players (in this book players are ravens adorably illustrated in graphics by Malte Meinhausen). Each player has a blue or red hat on his head. They see the other players but they cannot see the colour of their own hat. All players have to guess their own hat's colour (red, blue, or don't know) without communicating with each other. The players, as a team, win the game if at least one player is correct and none is wrong (a don't know answer is never wrong). There exist many variations of this type of game which are called "hat-problems". The present problem asking for a collective strategy was originally formulated by Todd Ebert in 1998. It was Elwyn Berlekamp who later connected the solution to coding theory and solved it for n players where n is of the form $2^m - 1$. The solution for general n is still open today. This kind of (brief) historical background of the problem is also given for the problems of the other chapters, which is a nice feature of the book. To work towards solving the problem, first some guessing strategies or naive trials are proposed, possibly introducing a first formalism like the red-blue colours are coded as 0-1 and the configuration of three hats can be coded as a binary number from 000 to 111. The first bit is for player 1, the second for player 2, and the last for player 3. This shows that there are 8 possible states. Each player knows 2 of the 3 bits in that number and has to guess the bit at her own position. The probability of winning for the 3 players is maximized if each player chooses her own colour (0-1 bit) so that the "distance" to one of the $8 = 2^3$ possibilities is minimal. The crux is to define this distance as the Hamming distance, which is the mathematical contribution to the solution in the form of coding theory. Once the principle is clear, it is easy to generalize the solution to $n = 2^m - 1$ players.

This problem involves some elementary probability, and that is also an ingredient for several of the other problems discussed in this book (both authors are professors of statistics). Several variants of the game correspond to the following description. There is a set of players that have to guess something on the basis of partial information that is available to them and the goal is to agree on some strategy that will maximize their chance to win the game as a team. For example in the second game of this book, the n players have their name hidden in n boxes and they have to find the box with their name in a minimal number of trials for the whole team. Here the mathematical tool is the factorization of a permutation in cycles. In the existing literature, the players are often prisoners that are collectively freed if they win. In this book the characters in the story are varying, but all the illustrations stick to ravens with a hat.

Let me skim more quickly over the other chapters. Somewhat related to the two hat-problems that I mentioned before is a problem where there are more colours for the hats and where the players are lined up in such a way that each player can only see the other players and their hats that are in front of them. This seems less information than when they can all see each other, but when the players start guessing the colour of their own hat one after another, starting with the one at the back, this is extra information. Here cyclic groups come to the rescue to solve the problem. Another game is a magical card trick which is based on understanding correctly how much randomness is introduced in an ordered card deck by riffle shuffling or cutting the deck. While the previous problems involved simple counting, a bit of probability, and basic algebraic concepts, there is no way around introducing the logarithm when information theory is involved. Not that the logarithm is an advanced topic, but still it requires some understanding of mathematics beyond counting. A further game introduces some projective geometry with a pinch of graph theory. Somewhat of a different nature is the problem involving two globes placed randomly on the table. Find the place or country on these globes that are in the same (or opposite) position on the two globes. This means that the axis defined by the position of that place on globe 1 and the globe centre should be parallel to the corresponding axis for globe 2. Here it should be realized that there is a mathematical one-to-one match of the two globes by a translation (matching the centres) and a single rotation. The axis of the latter gives the solution. This requires the introduction of some linear algebra to see that the problem is a simple eigenvalue problem for a 3D rotation matrix. The last game is again a classic and involves the hanging of a picture on the wall. If there is one nail on which the frame string can be hooked, then the picture will fall when the nail is removed. The game is to wind the chord of the frame around two (or more) nails such that if a nail is removed, the frame will not drop to the floor. Here the naive reader is confronted with group theory and algebraic topology to find a general solution.

For an inexperienced puzzler, the problems look challenging at first sight, so she is gradually guided by the authors towards a solution with several variants of naive trials, pointing at the shortcomings, and just enough mathematics is used to deal with the problem at hand. If, in a top-down approach, the mathematics were introduced first with the problem solution as an a posteriori application, that would not have had the same motivating effect as how it is presented here. A simple problem, but with a hard solution, leads to the mathematics that not only solves the original problem, but that can immediately be applied to generalizations and other variants of the problem. For the readers who are attracted to solving puzzles and games, but who have a weak or no mathematical background, the authors provide several appendices with additional explanation of notation, binary and complex numbers, converging sequences, probability, etc. Also some extra details on specific problems discussed in the chapters can be found there. This is an engaging book that all puzzlers, and certainly novices to the field, will love.

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