Arnold F. NIKIFOROV, Vasilii B. UVAROV, *Special Functions of Mathematical Physics : A Unified Introduction with Applications*, Translated from the Russian by R.P. Boas. Birkhäuser Verlag, Basel-Boston, 1988, xviii + 427 pages, Hard Cover, FS 168.

This book is a translation from the Russian. The original book was published in 1978 by Science, Moscow. If a book is translated into English from a foreign language, then this preassumes a certain quality of the original. Otherwise, it would not be worthwile translating. As it is often the case, such a translation allows the authors to do some rearrangements, correct some misprints, improve the presentation, add some new material etc. This is also what happened here. Also, translating itself is more than a passive job. It happens that literal translations of certain names from one language are not the costumary names used in the other language. For example the Russian name used for Bessel functions is cylinder functions. Also names for classical results may be attributed to other persons. What we know in the Western mathematical literature as the Schwarz inequality is known as the Sovjet mathematical literature as the Cauchy-Bunyakovsky inequality. In fact Bunyakovsky's priority is unquestionable but it now happens to be the name Schwarz that was attached to it in the Western literature. In this book, the translator has actively contributed to the final product. The translator, Ralph P. Boas is professor emeritus of Mathematics of the Northwestern University, Evanston, Illinois. He was executive editor of Mathematical reviews from 1945-50 and currently managing editor of Selecta Mathematica Sovietica.

An introduction to mathematical physics is unthinkable without a study of special functions. The authors are specialists in mathematical physics and quantum mechanics which explains their expertise in the field. It also explains the choice of the applications they give in the last chapter. It goes without saying that the material included in this book is of interest for all physicists, applied or theoretical, applied mathematicians, engineers, numerical analysts etc.

The most commonly encountered special functions are the classical orthogonal polynomials (Jacobi, Laguerre, Hermite), spherical harmonics, Bessel and hypergeometric functions. There appeared many books on these functions. The purpose of this book is to give a general, but simple treatment of the subject, avoiding special devices and cumbersome mathematical techniques. The approach chosen by the authors to meet this objective turns out to be a generalization of the Rodrigues formula for classical orthogonal polynomials. This formula makes it possible to obtain explicit integral representations and to derive the basic properties of the special functions to be studied. In particular, this technique is used to solve differential equations avoiding Laplace transforms. A minimum of mathematical apparatus is of course unavoidable. One needs the elements of the theory of ordinary differential equations and of complex analysis. These differential equations are of course necessary since almost all the special functions are solutions of a differential equation (1) : $u'' + (\tilde{\tau}/\sigma)u' + (\tilde{\sigma}/\sigma^2)u = 0$ with $\sigma, \tilde{\sigma}$ polynomials of degree at most 2 and $\tilde{\tau}$ a

polynomial of degree at most 1.

The book is organized as follows. Chapter 1 discusses a class of transformation $u = \phi y$ which can transform the equation (1) to the simpler form (2) : $\sigma y'' + \tau y' + \lambda y = 0$ with τ a polynomial of degree at most 1 and λ a constant. The solutions of the latter, called functions of hypergeometric type, are then studied. The theory is developed gradually. It is shown first that the derivatives of these functions are again in the same class. This makes it possible to construct solutions corresponding to particular choices of λ. E.g., $y = const$ for $\lambda = 0$ and polynomials in general. Explicit forms are given by means of the Rodrigues formula. This formula is then generalized to find integral representations for hypergeometric type functions for arbitrary λ. This integral representation plus some transformations of the equation (2) into other equations of the same type now lead to all the basic properties of the functions in question. Eventually, the complete family of solutions of (1) is obtained. All this takes but 9 pages : quite an achievement.

The program sketched in chapter 1 is worked out for special cases in the next three chapters. These three chapters can be read in any order. Chapter 2 gives the classical orthogonal polynomials. These are certainly the most important special functions and that is why chapter two takes 180 pages which is almost half of the book. The treatment here also includes orthogonal polynomials in a discrete variable, i.e., they are orthogonal with respect to summation. The differential equations are replaced by difference equations. This is probably the first time that this theory of continuous and discrete orthogonal polynomials is completely worked out in the same volume. Of course orthogonality with respect to summation is not new but the nice parallel with the continuous case using differences instead of derivatives seems to be new. Topics treated in this chapter are : basic properties, recursion, Christoffel-Darboux formula, zeros, asymptotics, series expansion into orthogonal polynomial series, orthogonal polynomials as eigenfunctions, spherical harmonics (Laplace equation in spherical coordinates), functions of the second kind, polynomials in a discrete variable on a uniform lattice (Mahn, Chebyshev, Meixner, Kravchuk, Charlier polynomials), extensions to a uniform lattice.

Chapter 3 is devoted to a second important kind of special functions : the Bessel functions. This chapter has 50 pages. Besides the classical theory, a section on semiclassical approximation is included. This gives a solution to a problem of finding uniformly asymptotic solutions of certain differential equations appearing in quantum mechanics. This theory was developed by Wentzel, Kramer, Brillouin and Langer.

Chapter 4 treats then in more detail the general hypergeometric functions. This is a relatively short chapter of 40 pages.

The last chapter proposes some applications in mathematical physics, quantum mechanics and numerical analysis. These include : the reduction of partial differential equations to ordinary equations by the method of separation of variables, the solution of boundary value problems of mathematical physics (Sturm-Liouville, Fourier-Bessel

expansions ...), some basic problems in quantum mechanics (Schrödinger, Klein-Gordon, Dirac equations, discrete polynomials), quadrature, information compression by discrete polynomials and applications of Bessel functions to laser sounding.

The treatment is relatively elementary. At regular places one can find tables which summarize the formulas for the different cases. A relatively extensive appendix contains a list of basic formulas. All this makes the book useful as a textbook as well as an easy to use reference work. Readers that lack the necessary background can find in an appendix an elementary treatment of the gamma function and some properties of the Laplace integral. Furthermore a list of tables, figures, symbols, an index and a relatively short bibliography are given.

The format used by Birkhäuser is the same as for another translation of a Russian standard work by Samarskii and Nikolaev on *Numerical Methods for Grid Equations*. (see G288a,b). The quality of contents and presentation is equally high.

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