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Control design for time-delay systems based on quasi-direct pole placement

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ABSTRACT

A novel method for the determination of controller parameters in a broad class of linear control systems affected by time-delays is presented. This method is based on an appropriate shaping of the spectrum of the closed-loop system. Its application follows two steps. First, a number of rightmost poles, smaller than the number of controller parameters, are directly assigned. This leads to constraints on the controller parameters. By using algebraic techniques a complete parametrization of all controllers satisfying these constraints is obtained. In the second step, the remaining degrees of freedom in the parameter space are used to shift the remaining part of the system spectrum as far to the left as possible. This corresponds to an optimization problem involving a nonsmooth, non-convex objective function. An extensive real-plant example is presented to demonstrate the effectiveness and applicability of the procedure.

1. Introduction

In this article we present a general method for the computation of controller parameters in control systems with time-delays, which is inspired by the classical pole placement method for systems without delays. For nth order SISO systems, the pole placement method allows the assignment of n poles to desired positions and, accordingly, the gain values of a state feedback controller are computed, see e.g. [8]. As it has been shown in [21,22,10,18], the same idea can be applied to adjust the dynamics of time-delay systems. However, such a direct assignment of poles has considerable limitations induced mainly by the infinite system's spectrum and the limited degrees of freedom in the controller parameter space.

We consider a general retarded system of the form

$$\dot{x}(t) = \int_{-T}^{0} dA(\tau)\, x(t+\tau) + \int_{-T}^{0} dB(\tau)\, u(t+\tau), \tag{1}$$

where $x \in \mathbb{R}^n$ is the vector of state variables, $u \in \mathbb{R}$ is the system's input, τ is the delay variable, which is constrained by the relation $0 \leqslant \tau \leqslant T$. The functional matrices

$$\tau \mapsto A(\tau) \in \mathbb{R}^{n\times n}, \quad \tau \mapsto B(\tau) \in \mathbb{R}^{n\times 1} \tag{2}$$

have bounded variation on $[-T,\ 0]$ and satisfy $A(0) = \mathbf{0}$ and $B(0) = \mathbf{0}$. They describe the distribution of the delay and cover both multiple lumped (pointwise) and distributed delays, [6,21]; for example, discontinuities of (2) lead to the presence of lumped delays. We consider a feedback controller of the form

$$u(t) = \sum_{j=1}^{p} k_j \left(\int_{-T}^{0} dC_j(\tau)\, x(t+\tau) \right), \tag{3}$$

where

$$K := [k_1\ k_2\ \ldots\ k_p]^T$$

contains the controller parameters to be determined. Once again, the functions $\tau \mapsto C_j(\tau)$ have bounded variation on $[-T,0]$ and satisfy $C_j(0) = \mathbf{0},\ j = 1,\ldots,p$.

We note that a broad class of systems and controllers lead to a closed-loop equation of the form (1) and (3). The class of controllers includes for instance conventional state feedback $u(t) = K^T x(t)$, delayed state and output feedback, using one or multiple delayed measurements, e.g. $u(t) = \sum_{i=1}^{q} K_i^T y(t-\theta_i)$.

The stability properties of the feedback system (1) and (3) are determined by the roots of the characteristic equation

$$\det\left(\lambda I - A(\lambda) + \sum_{j=1}^{p} k_j B(\lambda) C_j(\lambda) \right) = 0, \tag{4}$$

where

$$\begin{aligned} A(\lambda) &= \int_{-T}^{0} \exp(\lambda\tau) dA(\tau), \\ B(\lambda) &= \int_{-T}^{0} \exp(\lambda\tau) dB(\tau), \\ C_j(\lambda) &= \int_{-T}^{0} \exp(\lambda\tau) dC_j(\tau), \quad j = 1,\ldots,p. \end{aligned}$$

The system (1) and (3) is exponentially stable if and only if all the roots of Eq. (4) are located in the open left half plane, [7,12]. As

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the time-delay system is retarded, the distribution of the (infinitely many) characteristic roots has the following properties.

Property 1. *For any $\beta \in \mathbb{R}$, the number of roots of (4) in the half plane $\Re(\lambda) > \beta$ is finite*, [12].

Property 2. *For $|\lambda_i| > c \gg 0$ the roots of a retarded system tend to match a finite number of exponential curves spreading out in the left half plane*, [1,19].

A consequence of Property 1 is that a retarded system can have finitely many unstable characteristic roots only. A consequence of Property 2 is that infinite chains of roots converge to $-\infty$ in real and to $\pm\infty$ in imaginary part. As a consequence of both properties, the dynamics of a retarded system are determined by a small number of rightmost characteristic roots, [21].

Following the lines of [21,22], generically p system poles can be placed to desired positions by p controller parameters. However, such a placement is effective if and only if the remainder of the system's spectrum is located to the left of these assigned dominant poles, which is not guaranteed. In fact, the direct pole placement relies on a rather heuristic trial and error placement of the dominant poles. If the assigned poles are not isolated from the remainder of the spectrum, the pole placement procedure needs to be repeated with a different selection of the poles.

A semi-automated pole placement based control design – *continuous pole placement* – has been introduced in [10] for retarded systems and in [13] for neutral systems. The principle of this method is to shift the rightmost roots of the feedback system as far to the left as possible via a step-by-step shifting of a few characteristic roots, while monitoring the behavior of the other rightmost roots. In this way, the spectral abscissa,

$$\alpha := \sup_{\lambda \in \mathbb{C}} \{\Re(\lambda) : \ \lambda \text{is a characteristic root}\}, \tag{5}$$

is minimized. More recently, a direct optimization approach has been used to minimize the spectral abscissa in [18]. As the spectral abscissa is a nonsmooth function, the gradient sampling algorithm was used, see [3,4].

Even though the minimization of the spectral abscissa α is very useful from a stabilization point of view, it is in fact not a true pole placement method.

Although the resulting dynamics are the fastest possible, there is no guarantee of proper damping of the system's modes as the imaginary parts of the poles are not controlled.

In this paper, we provide a method for determining the controller parameters which combines the positive features of a direct pole placement on the one hand and a minimization of the spectral abscissa function on the other hand. More precisely, the method consists of assigning a *small number* of system poles and pushing the remainder of the spectrum as far to the left as possible by utilizing the remaining degrees of freedom in the parameter space. The aim of this combined approach is to guarantee that the assigned poles will be the dominant poles of the feedback system and will determine the system dynamics.

2. Stabilization with pole location constraints

Under the condition

$$\det(\lambda I - A(\lambda)) \neq 0$$

the characteristic equation can be written as

$$\det\left(I - \sum_{j=1}^{p} k_j B(\lambda) C_j(\lambda)(\lambda I - A(\lambda))^{-1}\right) = 0$$
$$\Longleftrightarrow 1 - \sum_{j=1}^{p} C_j(\lambda)(\lambda I - A(\lambda))^{-1} B(\lambda) k_j = 0.$$

Assigning a real pole to the location c yields the following constraint on the gain values:

$$\sum_{j=1}^{p} C_j(c)(cI - A(c))^{-1} B(c) \ k_j = 1.$$

Similarly, assigning a complex conjugate pair of poles, $c \pm di$, results in

$$\begin{cases} \sum_{j=1}^{p} \Re\left\{C_j(c+di)((c+di)I - A(c+di))^{-1} B(c+di)\right\} k_j &= 1, \\ \sum_{j=1}^{p} \Im\left\{C_j(c+di)((c+di)I - A(c+di))^{-1} B(c+di)\right\} k_j &= 0. \end{cases}$$

In this way, assigning $0 \leqslant m \leqslant p$ poles to $\lambda_1, \ldots, \lambda_m$ eventually results in a set of m constraints which can be written in the form

$$SK = R, \tag{6}$$

where $S \in \mathbb{R}^{m \times p}$ and $R \in \mathbb{R}^{m \times 1}$.

If the pole assignment problem is solvable, that is, Eq. (6) has solutions, then the latter describes a *linear subspace in the parameter space* corresponding to the presence of the characteristic roots $\lambda_1, \ldots, \lambda_m$ in the spectrum of the feedback system (1) and (3).

Let us simplify this subspace by using another basis of the parameter space. Using the singular value decomposition $S = U\Sigma V^*$, where $(\cdot)^*$ denotes the complex conjugate transpose, condition (6) becomes

$$\Sigma L = U^* R \tag{7}$$

where

$$L = V^* K. \tag{8}$$

Letting $L = [l_1 \ \cdots \ l_p]^T, U^*R = [\bar{r}_1 \ \cdots \ \bar{r}_m]^T, \Sigma = [\mathrm{diag}(\sigma_1, \ldots, \sigma_m) \ \mathbf{0}]$, and assuming that S is of full (row) rank, we finally get the following expression for the parameters $l_j, \ j = 1, \ldots, m$, corresponding to the assigned poles $\lambda_i, \ i = 1, \ldots, m$:

$$\begin{cases} l_1 &= \bar{r}_1/\sigma_1, \\ l_2 &= \bar{r}_2/\sigma_2, \\ \vdots & \quad \vdots \\ l_m &= \bar{r}_m/\sigma_m. \end{cases} \tag{9}$$

The following can be concluded.

Proposition 3. *Consider the feedback system (1) and (3). By assigning m system poles $\lambda_1, \ldots, \lambda_m$, the system of m Eq. (6) is obtained, constraining the feedback gain K. If S has full row rank, then the constraint (6) is equivalent to (9), where the relation between K and $L = [l_1 \ \cdots \ l_p]^T$ is described by*

$$L = V^* K, \quad K = VL. \tag{10}$$

Theorem 4 (Controller parametrization). *Assume that S has full row rank. Considering parameters $l_1, \ldots, l_m$ as fixed according to (9), and $l_{m+1}, \ldots, l_p$ being kept as free parameters, one obtains a parametrization of all controllers that assign m poles to $\lambda_1, \ldots, \lambda_m$.*

The next step of the pole placement procedure is as follows. Assume that m poles are assigned as described in Proposition 3 and Theorem 4. Thus, the parameters $l_1, \ldots, l_m$ are fixed and the parameters $l_{m+1}, \ldots, l_p$ are available for further adjustment of the system's spectrum. Applying the algorithm described in [18], the parameters $l_{m+1}, \ldots, l_p$ are to be used to push the other rightmost poles as far as possible to the left. However, unlike in [18], where the spectral abscissa (5) is minimized, we minimize instead the function

$$\alpha(l_{m+1},\ldots,l_p) = \sup\left\{\Re(\lambda): \frac{\det\left(\lambda I - A(\lambda) - \sum_{j=1}^{p} k_j(l_{m+1},\ldots,l_p)B(\lambda)C_j(\lambda)\right)}{\Pi_{j=1}^{m}(\lambda-\lambda_j)} = 0\right\}. \quad (11)$$

Technically, the evaluation of $\bar{\alpha}$ is rather straightforward. First, the rightmost characteristic roots of (1) and (3) are computed, either directly by the quasi-polynomial rootfinder recently presented in [19], or, by solving an infinite-dimensional eigenvalue problem (e.g. using DDE-BIFTOOL, [5]). In the next step, the *invariant* characteristic roots $\lambda_1,\ldots,\lambda_m$ are removed from the spectrum and the objective function is obtained as $\bar{\alpha} = \sup_{i>m}(\Re(\lambda_i))$. Similarly as in [18], it can be shown that the function $\bar{\alpha}$ is smooth almost everywhere (i.e., except for a set in the parameter space with measure zero). More precisely, the function is smooth whenever there is only one non-assigned characteristic root whose real part is equal to $\bar{\alpha}$. In such a case one can compute the gradient of $\bar{\alpha}$ from the sensitivity of this characteristic w.r.t. the controller parameters (in this context, see Chapter 3 of [12]).

The gradient sampling algorithm, presented in [3] and used in [18] can now be applied to minimize the function

$$(l_{m+1},\ldots,l_p)\mapsto\bar{\alpha}(l_{m+1},\ldots,l_p). \quad (12)$$

This algorithm, developed for the minimization of nonsmooth, nonconvex functions, which are differentiable almost everywhere is essentially the same as the *Steepest Descent* method, apart from the fact that an approximation of the nonsmooth steepest descent direction is used. The latter can be defined as

$$-\arg\min_{z\in\partial_c\phi(\bar{L}_c)}\|z\| \quad (13)$$

where $\partial_c\phi(\bar{L}_c)$ denotes the generalized gradient (referred to as the *Clarke Subdifferential*) at $\bar{L}_c$, where $\bar{L}_c = [l_{m+1},\ldots,l_p]$. It is given by

$$\partial_c\phi(\bar{L}_c) = \mathrm{conv}\left\{\lim_{\bar{L}\to\bar{L}_c}\vec{\nabla}\bar{\alpha}(\bar{L})\right\}, \quad (14)$$

where $\mathrm{conv}(\cdot)$ denotes the convex hull. Note that the generalized gradient reduces to the classical notion of gradient whenever the function is differentiable. In the gradient sampling algorithm the Clarke subdifferential is approximated by sampling gradients in a neighborhood of $\bar{L}_c$ and considering the convex hull.

An application of the algorithm leads to a monotonic decrease of the objective function until a local minimum is reached. Such a minimum is characterized by $\vec{0}\in\partial_c\phi(\bar{L}_c)$.

In order to speed up the optimization process we start by applying the BFGS algorithm to (12), which performs surprisingly well on nonsmooth problems (see [9] for a detailed analysis), and switch only to the gradient sampling algorithm on the moment that the BFGS algorithm stagnates.

The pole placement procedure can now be summarized as follows:

Algorithm 5 (*Quasi-direct pole placement*). Consider feedback system (1) and (3).

1. Select poles $\lambda_1,\ldots,\lambda_m$, $m<n$ to be assigned.
2. Compute the parameters $l_1,\ldots,l_m$ as described in Proposition 3.
3. Minimize the function (12).
4. If $\min\bar{\alpha} < \min_{1\leqslant j\leqslant m}\Re(\lambda_j)$, accept the result and transform L to K by (10). In the other case (that is, the assigned poles cannot be separated from the remainder of the spectrum), select different assigned poles and go to step (1).

In the application of the algorithm the following guidelines are useful.

- With the choice of the assigned poles the user can assign the speed of the responses, damping of the oscillatory modes, etc., provided that the assigned roots can be sufficiently isolated from the remainder of the spectrum. It is advised to assign only a small number of poles, not only because it facilitates their choice, but also because this leaves much degrees of freedom in the controller parameter space that can be used to enforce the spectral separation.
- In order to select realistic pole locations that take into account the difficulty of the problem and the constraints induced by the delays, the spectral abscissa (5) can be minimized over *all* available controller parameters in the first place, as it is described in [18]. It is then advised to assign the poles $\lambda_j,\ 1\leqslant j\leqslant m$, such that

$$\min_K\alpha(K) < \min_{1\leqslant j\leqslant m}\Re(\lambda_j). \quad (15)$$

  Indeed, note that if the rightmost assigned pole has real part smaller than $\min\alpha(K)$, then the assigned poles cannot be the rightmost ones.
- A good starting value for the optimization problem in Step 3. of the algorithm can be obtained from

$$\arg\min_K\{\|K-K_0\|:\ SK=R\}, \quad (16)$$

  where $K_0 = \arg\min\alpha(K)$. In words, the expression (16) corresponds to the gain value in the subspace (6) closest to K_0. In this way, it is expected that the local minimizer of $\bar{\alpha}$, found by step (3) of Algorithm 5, also corresponds to a gain value close to K_0. This is important to prevent a large qualitative change in the uncontrolled part of the spectrum, which we want to avoid because the assigned poles are aimed to be the rightmost. An explicit computation of (16) leads to

$$\begin{bmatrix} l_{m+1}\\ \vdots\\ l_p\end{bmatrix} = [v_{m+1}\ldots v_p]^{\dagger}\left(K_0 - [v_1\ldots v_m]\begin{bmatrix} l_1\\ \vdots\\ l_m\end{bmatrix}\right), \quad (17)$$

  where $V=[v_1\ldots v_m]$, $l_1,\ldots,l_m$ are given by (9), and $(\cdot)^{\dagger}$ denotes the Moore–Penrose (generalized) inverse.

Remark 6. The method can be extended to or made more robust against time-varying delays or perturbations on other system's parameters. If the variation is slow compared to the system's dynamics and can be measured or estimated on-line, this information can be incorporated by working with a gain-scheduled controller that interpolates between a limited number of precomputed sets of gain values- the closed-loop system would become a so-called LPV system [2]. If the parameter variation is fast or only bounds are known on its value and/or its derivatives, one can enforce robustness against the parametric uncertainty by adding a term to the objective function which measures the robustness of stability against the time-varying uncertainty, e.g. the reciprocal of a stability radius as defined in [11]. Note that the presented method offers the flexibility to adapt the objective function or to add constraints. All these issues are however beyond the scope of this article.

3. Application example – experimental heat transfer set-up

The above control design algorithm has been applied to the mathematical model of the experimental heat transfer set-up, which has been described comprehensibly and identified in [20].

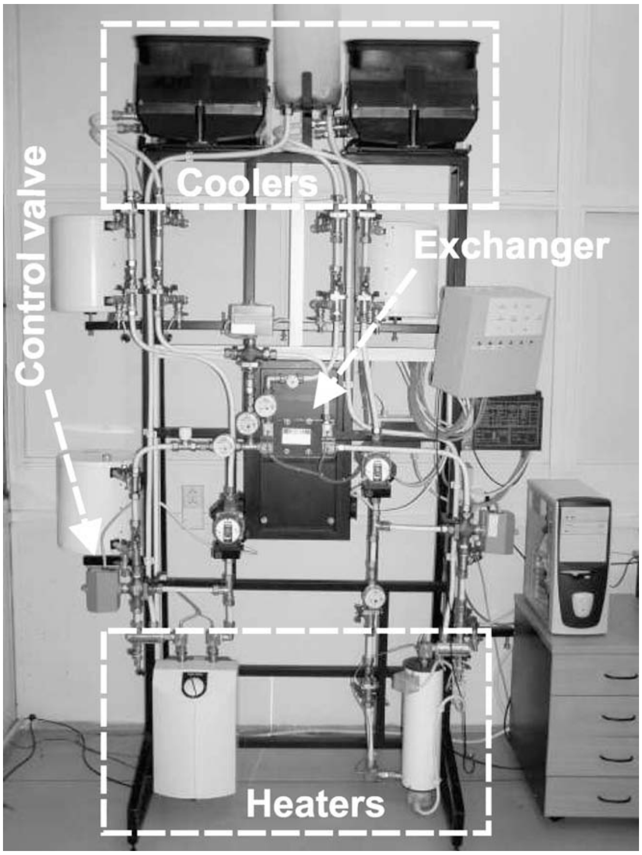

Fig. 1. Experimental heat transfer set-up with heat exchangers.

The set-up, which is shown in Fig. 1, consists of two closed heating circuits (left and right) with forced circulation of water as the heat transfer medium. Both circuits are equipped with heaters and coolers. The heat exchange between the circuits takes place in a heat exchanger. These main subsystems of the set-up are connected by pipe-lines. Obviously, the ratios between the lengths of pipe-lines and flow velocities determine the main transport delays in the heat transfer throughout the system. Besides, the set-up is equipped with three mixing valves for controlling the flows in some parts of the set-up. The control inputs of the system are the performances of heaters and coolers and the system outputs are temperatures measured at inputs and outputs of the main subsystems.

In [20], a linear model of the set-up is proposed, which consists of ten linear differential equation of the form (1). The corresponding matrices are given by $A(\lambda) = [a_{i,j}]$ and $B(\lambda) = [b_{i,k}], i,j = 1 \dots 10, k = 1,2,$ where $a_{1,1}(\lambda) = -\frac{1}{5}$, $a_{1,6}(\lambda) = \frac{0.973\exp(-\lambda 15)}{5}$, $a_{2,1}(\lambda) = \frac{0.96\exp(-\lambda 23)}{25}$, $a_{2,2}(\lambda) = -\frac{1}{25}$, $a_{3,2}(\lambda) = \frac{0.983\exp(-\lambda 5)}{5}$, $a_{3,3}(\lambda) = -\frac{1}{5}$, $a_{4,3}(\lambda) = \frac{0.575}{3}$, $a_{4,4}(\lambda) = -\frac{1.425}{3}$, $a_{4,7}(\lambda) = \frac{0.85}{23}$, $a_{4,8}(\lambda) = \frac{0.85}{23}$, $a_{5,4}(\lambda) = \frac{0.975\exp(-\lambda 3)}{5}$, $a_{5,5}(\lambda) = -\frac{1}{5}$, $a_{6,5}(\lambda) = \frac{0.9\exp(-\lambda 5)}{17}$, $a_{6,6}(\lambda) = -\frac{1}{17}$, $a_{7,7}(\lambda) = -\frac{\exp(-\lambda 29)}{63}$, $a_{8,3}(\lambda) = \frac{0.85}{23}$, $a_{8,4}(\lambda) = \frac{0.85}{23}$, $a_{8,7}(\lambda) = \frac{0.575}{3}$, $a_{8,8}(\lambda) = -\frac{1.425}{3}$, $a_{9,8}(\lambda) = \frac{0.97\exp(-\lambda 5)}{5}$, $a_{9,9}(\lambda) = -\frac{1}{5}$, $a_{10,9}(\lambda) = \frac{0.92\exp(-\lambda 5)}{15}$, $a_{10,10}(\lambda) = -\frac{1}{15}$, $b_{7,1}(\lambda) = \frac{\exp(-\lambda 7)}{63}$, $b_{10,2}(\lambda) = \frac{0.042\exp(-\lambda 6)}{15}$, while the other elements in the matrices $A(\lambda)$ and $B(\lambda)$ are equal to zero. Note that this corresponds to a state space representation involving five different delays in the state (namely 3,5,15,23 and 29 s) and two different delays in the inputs (6 and 7 s). In order to demonstrate how the various state variables are coupled and which of the couplings are delayed, we provide the information on the structure of the matrices $A(\lambda), B(\lambda)$ as follows:

$$\begin{bmatrix} 0 & . & . & . & . & 15 & . & . & . & . \\ 23 & 0 & . & . & . & . & . & . & . & . \\ . & 5 & 0 & . & . & . & . & . & . & . \\ . & . & 0 & 0 & . & . & 0 & 0 & . & . \\ . & . & . & 3 & 0 & . & . & . & . & . \\ . & . & . & . & 5 & 0 & . & . & . & . \\ . & . & . & . & . & . & 29 & . & . & . \\ . & . & 0 & 0 & . & . & 0 & 0 & . & . \\ . & . & . & . & . & . & . & 5 & 0 & . \\ . & . & . & . & . & . & . & . & 5 & 0 \end{bmatrix}, \quad \begin{bmatrix} . & . \\ . & . \\ . & . \\ . & . \\ . & . \\ . & . \\ 7 & . \\ . & . \\ . & . \\ . & 6 \end{bmatrix},$$

where the numbers represent the values of the delays at nonzero elements, while the dots represent the zero elements.

The system's state variables are the temperatures measured at different places of the set-up. We consider here only two out of four system inputs from [20], $u = [u_c, u_d]^T$. The first – the control signal u_c – is the set-point value of the slave PI control loop that controls the performance of the left heater (the slave control loop is included in the model of the set-up). The second signal u_d adjusts the performance of the left cooler and is considered here as a disturbance.

The controlled variable y is the temperature measured at the output of the left cooler, which is the last state variable in the vector x. The feedback controller is considered of mixed PI/state feedback type. Is has the form

$$u_c(t) = -K^T \bar{x}(t) \tag{18}$$

where $\bar{x}(t)) := [x(t)^T I(t)]^T$ is the extended vector of state variables and

$$I(t) := \int_0^t (y_{SET}(\eta) - y(\eta))\, d\eta \tag{19}$$

is an additional state variable introduced for control purposes, y_{SET} being the set-point of y. In Fig. 2, the rightmost poles of the open loop system are shown. As can be seen, due to the presence of the integral (19), one pole is placed at the complex plane origin. The remaining poles determine the modes of the system.

The parameters of the feedback gain K have been determined by applying steps 1–3 of the quasi-direct pole placement procedure

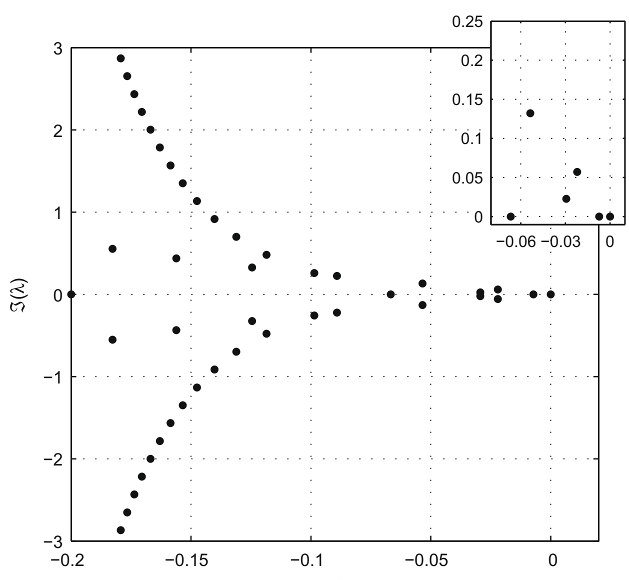

Fig. 2. Spectrum of the open loop system.

described in Algorithm 5. To illustrate the effectiveness and applicability, we have considered different cases where up to four eigenvalues are assigned. The results are presented in Table 1. As can be seen, no poles are assigned for the setting SN 1, which corresponds to the minimum of the spectral abscissa (5) over all 11 parameters of the feedback gain K. The resulting spectrum for this setting is shown in Fig. 3. As explained in the guidelines, it is preferred to assign poles to the right of $\min\alpha = -0.0565$. Based on this we have chosen three sets of poles to be assigned. First, only a single pole was assigned at $\lambda_1 = -0.01$ resulting in $\min\bar{\alpha} = -0.0629$ (SN 2 in Table 1, see the spectrum in Fig. 4). Secondly a pair of poles $\lambda_{1,2} = -0.02 \pm 0.02i$ were assigned resulting in $\min\bar{\alpha} = -0.0659$ (SN 3 in Table 1, see the spectrum in Fig. 5). Finally, four poles were assigned as listed in the last column of Table 1. This resulted in $\min\bar{\alpha} = -0.0736$ and the spectrum shown in Fig. 6.

In Fig. 7 the computed set-point and disturbance responses corresponding to the different settings are compared. The responses of the uncontrolled system are slow, while the disturbance creates an off-set. The latter is removed by including integral action in the controller. As expected, the responses for SN 1 are the fastest, because this setting corresponds to a minimum of the spectral abscissa. Note that the overshoots are very high accordingly. For SN 2 the responses are fairly slow. SN 3 leads to fast and well-damped responses, however, with a considerable overshoot. The responses of SN 4 are the best achieved – fast, well-damped and with small overshoots. For the demonstration purposes, we describe in more detail the control synthesis for SN 4, where $m = 4$ poles are assigned. First, the pole constraints are expressed in the form (6) and the transformed parameters $l_j,\ j = 1,\ldots,4$ are obtained from (9). The remaining parameters are computed by minimizing (12), with starting values (17). Iterations of the optimization algorithm are shown in Fig. 8. At iteration number 105, indicated with a dashed line, the algorithms switches from BFGS to gradient sampling. Finally, the controller gain K is obtained from the transformation (10).

The above results clearly show the effectiveness of the new pole placement design method for the heat transfer setup model, having a rather involved structure with several distinct delays in both

Table 1
Results of the quasi-direct pole placement algorithm. SN – setting number; λ_i – assigned poles; $\min\bar{\alpha}$ – minimum of (12); $k_j,\ j = 1,\ldots,11$ – resulting elements of the gain matrix K in (18).

SN	1	2	3	4
λ_i	–	−0.01	−0.02 ± 0.02i	−0.02, −0.03 −0.03 ± 0.03i
$\min\bar{\alpha}$	−0.0565	−0.0629	−0.0659	−0.0736
k_1	−5.4349	−0.0732	−4.1420	−0.3521
k_2	3.5879	8.1865	5.9345	8.6190
k_3	−1.4411	−1.2503	−2.3820	−4.8822
k_4	−3.7043	−7.1472	−7.9449	−17.2747
k_5	24.616	32.8003	27.8585	35.1494
k_6	−2.1778	4.4977	0.4490	−1.3188
k_7	9.6924	10.3140	8.4887	6.0338
k_8	−4.5121	−2.6572	−0.2605	5.4190
k_9	−14.631	−21.6711	−20.5152	−24.6596
k_{10}	11.351	4.1244	5.4531	2.3754
k_{11}	−0.7562	−0.2749	−0.3635	−0.1360

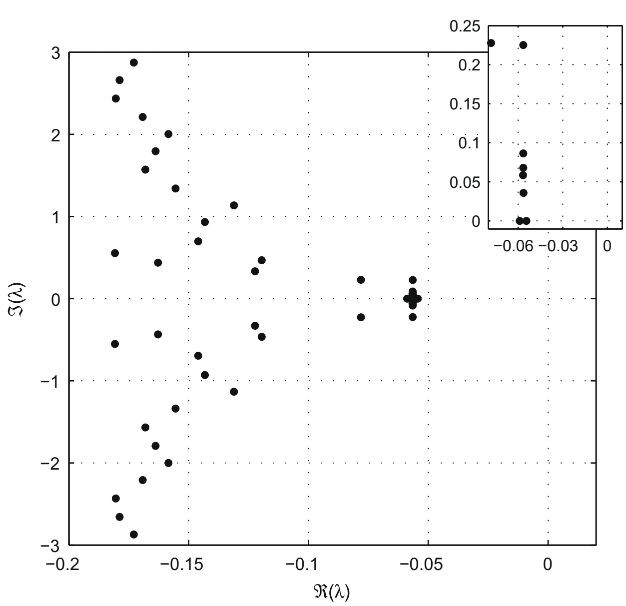

Fig. 3. Spectrum of the feedback system for SN 1 in Table 1.

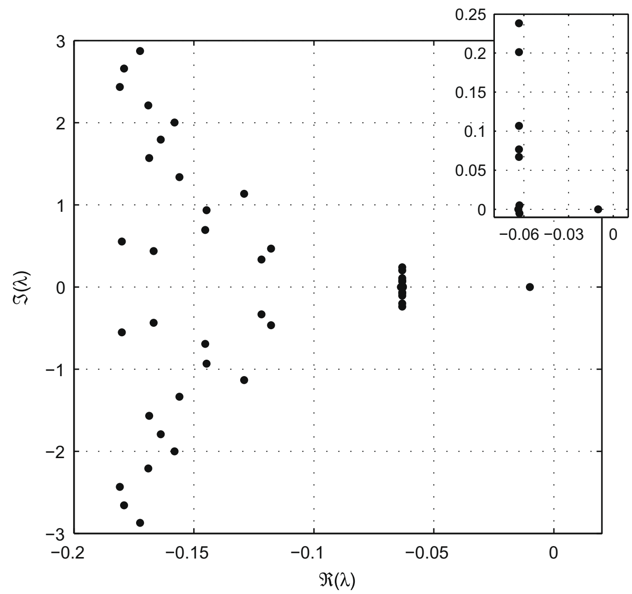

Fig. 4. Spectrum of the feedback system for SN 2 in Table 1.

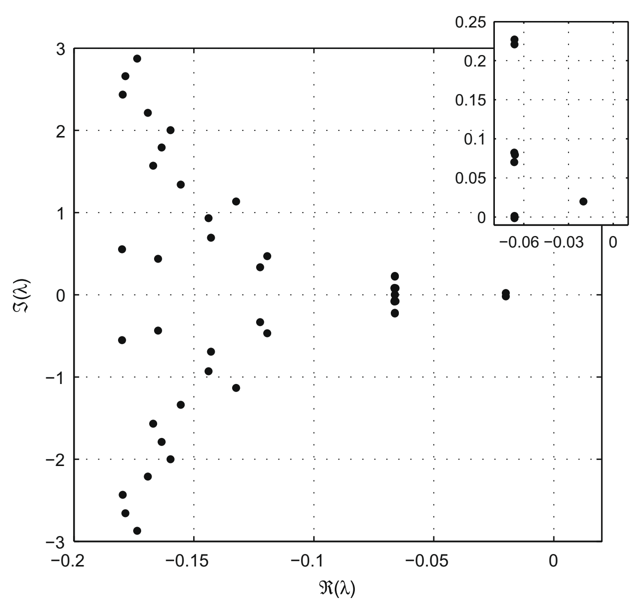

Fig. 5. Spectrum of the feedback system for SN 3 in Table 1.

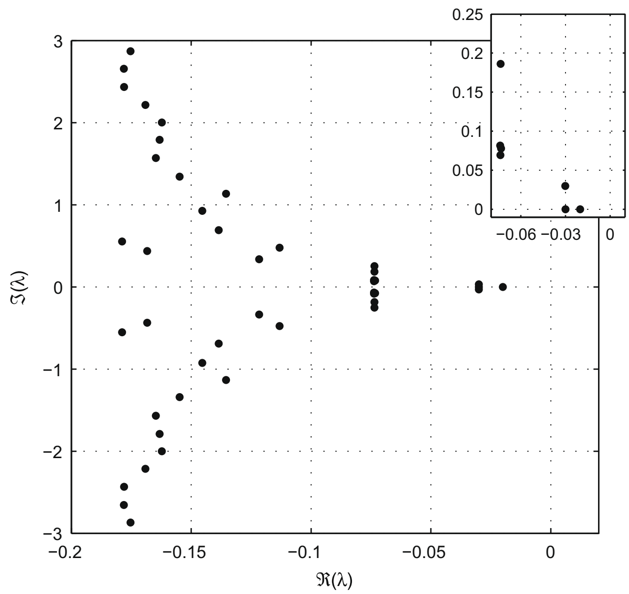

Fig. 6. Spectrum of the feedback system for SN 4 in Table 1.

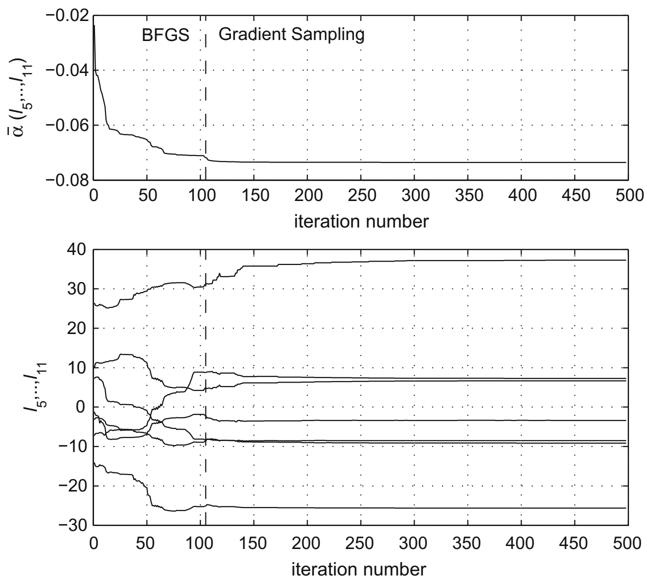

Fig. 8. The objective function and the variables $l_j,\ j = 5, \ldots, 11$ along the iterations of the BFGS/gradient sampling algorithm, for the setting given in SN 4 in Table 1.

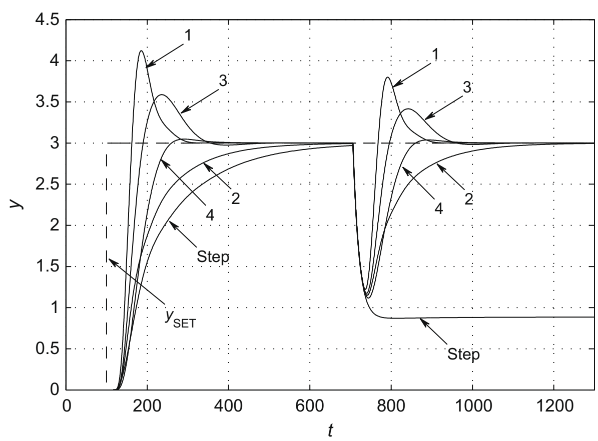

Fig. 7. Computed step responses of the uncontrolled system, for $u_c = 4$ at $t = 100$ and $u_d = 0.5$ at $t = 700$ (Step). Setpoint and disturbance rejection responses of the controlled system for $y_{SET} = 3$ at $t = 100$ and $u_d = 0.5$ at $t = 700$ (other curves). The curves are labeled by numbers corresponding to the settings described in Table 1.

states and inputs. Let us also mention that a considerable advantage of using the control law in the form (18) lies in the fact that it can be easily implemented, just by feeding back the measured signals to the control action. In the next stage, an experimental validation of the obtained results will be performed. Besides, the designed control technique will be compared with other types of control approaches such as model predictive control (MPC), which has proved very useful in many engineering applications, see e.g. [14] or [15–17].

4. Conclusions

A novel method for determining (controller) parameters in retarded time-delay systems has been proposed. It combines direct pole placement and the minimization of the spectral abscissa, reaping the benefits of the advantages of both approaches. The aim is to place a small number of distinct dominant poles determining the dynamics of the system. These poles should be placed sufficiently far from the imaginary axis to guarantee both the robust stability of the system and sufficiently fast dynamics. Besides, the ratio between real and imaginary parts of the poles should be large enough in order to guarantee a proper damping in the system responses. Further on, an optimization based pole shifting algorithm is used to push the remaining rightmost poles as far to the left as possible, because a pole distribution is only acceptable when the assigned poles are sufficiently isolated from the rest of the spectrum. In order to demonstrate its capabilities, the approach has been applied to the control synthesis of a mathematical model of the experimental heat transfer set-up. Besides the demonstration of the novel pole placement procedure, the contribution of the example section lies in a new application of optimization based fixed structure control design techniques.

Finally we note that the methodology of the paper is applicable to a very broad class of delay equations (including multiple discrete and distributed delays) and a very broad class of controllers (static, dynamic, delayed state and output feedback, PID, even controllers including particular types of predictors). The only main restriction is that the dynamics of the closed-loop system are described by a nonlinear eigenvalue problem, and that an algorithm to compute the corresponding rightmost (or dominant) characteristic roots is available.

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