DISC Course: Stability and Control of Time-Delay Systems

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Homework 3

We consider the heating system depicted in Figure 1. The uncontrolled system is modelled by the equations

$$\left\{\begin{array}{lcl} 14\dot{\theta}_h(t) & = & -\theta_h(t-6.5)+0.24\theta_a(t-40)+0.39\ u(t-13.2), \\ 3\dot{\theta}_a(t) & = & \theta_h(t)-2\theta_a(t)+\theta_c(t-13), \\ 3\dot{\theta}_d(t) & = & -\theta_d(t)+0.94\theta_a(t-18), \\ 25\dot{\theta}_c(t) & = & -\theta_c(t-9.2)+0.81\theta_d(t-2.8). \end{array}\right. \tag{1}$$

The state variables are the temperatures at various places, which are all accessible for measurements. The time-delays stem from modeling propagation through the pipes, and the input u affects the position of a valve. More details about the model can be found in [3].
The aim of the controller is to regulate the temperature θ_c to a desired set-point $\theta_{c,\mathrm{set}}$ and, more specifically, to minimize the response time to a change of the set-point. A combination of state feeeback and PI control is used:

$$u(t)=-k_1\theta_h(t)-k_2\theta_a(t)-k_3\theta_d-k_4\theta_c(t)-k_5 z(t), \tag{2}$$

where the internal state of the integrator, z, satisfies

$$\dot{z}(t)=\quad \theta_{\mathrm{c,set}}(t)-\theta_c(t), \tag{3}$$

see Figure 1.

Download and decompress the file "optim.zip" from

`http://www.cs.kuleuven.be/~wimm/disc.`

Next, follow the instructions, complete and execute the commands in the file "main.m". Notice that this file consists of three parts, which should be executed separately.

1. What is the purpose of the integral action in the control law?
2. Compute and plot the rightmost characteristic roots of (1)-(3) for $[k_1 \dots k_5] = [0\ 0\ 0\ 0\ 0]$. What is the rightmost characteristic root? Give an explanation? What can you say about the exponential decay rate of the solutions of the uncontrolled system?
3. Minimize the spectral abscissa function (real part of the rightmost characteristic root) as a function of the control parameters, $[k_1 \dots, k_5]$. What are the optimal gain values? Make a plot of the corresponding rightmost characteristic roots.

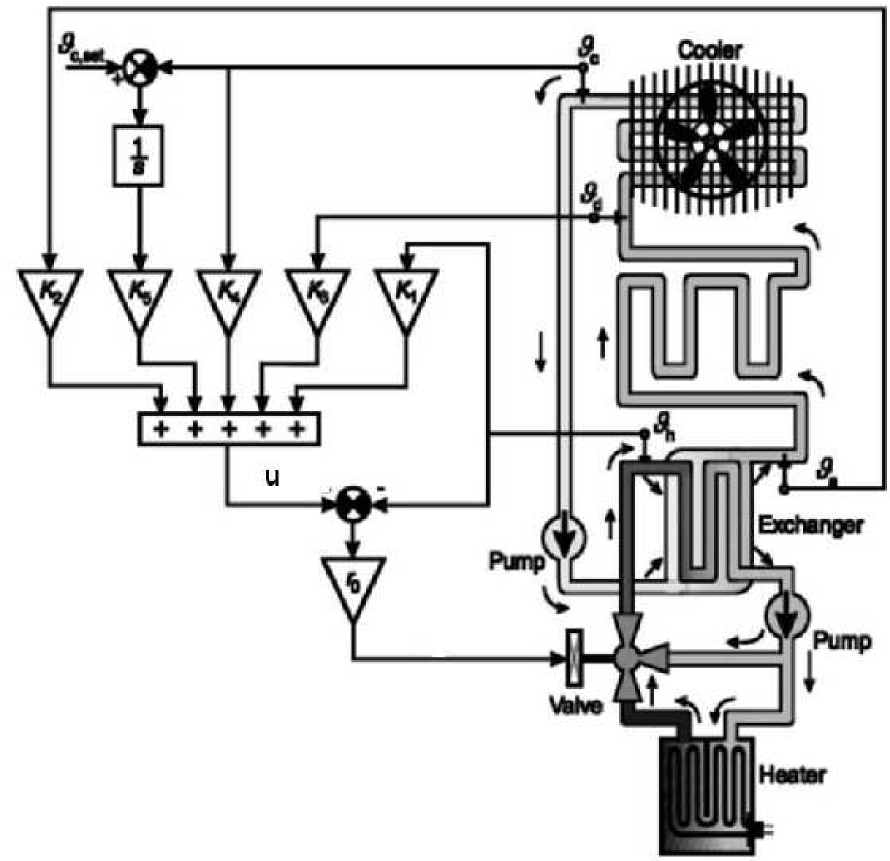

Figure 1: Diagram of the closed-loop system (1)-(2).

4. Do you recognize the second part of the file ”heattransfer.m”? What exactly is implemented there?
5. For the 'optimal'gain values investigate the *sensitivity* of the spectral abscissa to modeling errors on the input delay, by numerically computing the characteristic roots as a function of the input delay (part III of the file ”main.m”). Display your results. What do you conclude? Do you think that the observed phenomenon is generic? How could it be avoided?

You need to send me your homework by e-mail in two weeks (by December 10).

Good luck!

Acknowledgments

The software for the homework includes routines from the package DDE-BIFTOOL [2] and an implementation of the gradient sampling optimization algorithm, described in [1].

References

[1] J. V. Burke, A. S. Lewis, and M. L. Overton. A robust gradient sampling algorithm for nonsmooth, nonconvex optimization. *SIAM Journal on Optimization*, 15(3):751–779, 2005. An implementation is available at
`http://www.cs.nyu.edu/faculty/overton/papers/gradsamp/index.html`.

[2] K. Engelborghs, T. Luzyanina, and G. Samaey. DDE-BIFTOOL v. 2.00: a Matlab package for bifurcation analysis of delay differential equations. TW Report 330, Department of Computer Science, Katholieke Universiteit Leuven, Belgium, October 2001.

[3] Vyhlídal, T. *Analysis and synthesis of time delay system spectrum.* PhD thesis, Department of Mechanical Engineering, Chech Technical University at Prague, 2003.

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