Spectral properties of (perturbed) normal matrices and their applications

In this research project *computational and theoretical issues* linked with *spectral properties of the classes of normal and perturbed normal* matrices will be studied.

The overall research goal is *the development of new, theoretically well-supported, fast, accurate and reliable algorithms for computing (approximate) spectral decompositions of (perturbed) normal matrices*. The embracing goal encloses three weighty subgoals. We aim at notable progression in, and aspire towards significant contributions in *theory*, *algorithms design* and the *development of competitive software*.

These goals are translated into four work packages, each having clearly specified tasks to be tackled.

1. **Spectral computations (work package 1)** *In the first part of the project we focus on the design of new theoretical frameworks providing thereby substantial theoretical support for developing new algorithms for computing eigenvalues and singular values.* We consider
   (a) the construction of competitive algorithms for computing the singular value and eigenvalue decomposition of normal matrices supported by theoretical findings;
   (b) the development of new, alternative compact representations of normal matrices;
   (c) the search for the applicability of these novel algorithms to normal matrix pencils.
2. **Closest normal matrices (work package 2)** *In this project part we aim at developing new strategies for computing 'the closest normal matrix' or 'the closest slightly perturbed normal matrix' in a wide variety of senses.* We focus on
   (a) enhancing existing methods for computing the closest normal matrix;
   (b) designing new computational approaches for computing the closest normal matrix in various senses;
   (c) identifying the applicability of closest approximants, such as, e.g., preconditioners, solving nearby spectral problems,... .
3. **Structured rank relations (work package 3)** *In this work package we investigate how to exploit structured rank matrix techniques for dealing with normal matrices. We will try to maximally utilize low rank properties present in normal matrices reducing thereby the complexity of some spectral decomposition algorithms and obtaining for some cases an easier theoretical framework.* Here, we will strive to
   (a) identify the link with the Faber-Manteuffel theorem;
   (b) search for new structured rank based compact representations of normal matrices;
   (c) develop a framework for computing eigenvalues of perturbed normal matrices.
4. **High quality software (work package 4)** *This work package serves the scientific community by providing a tangible tool for the practitioner.* We consider:
   (a) experimental software, allowing the theoreticians to swiftly alter and test the routines;
   (b) a matrix test suite, enabling researchers to test novel techniques on a well-designed set of matrices;
   (c) state of the art software, where accuracy, speed and reliability are keywords. This results in software competitive with today's standards.