

# Logical Equivalences

Two sentences  $\alpha$  and  $\beta$  are **logically equivalent**, denoted  $\alpha \iff \beta$

iff they have the same set of models

iff  $\alpha \models \beta$  and  $\beta \models \alpha$

iff  $(\alpha \equiv \beta)$  is a tautology, i.e. true in every interpretation, i.e.  $\models \alpha \equiv \beta$

1. **idempotent law**  $(\alpha \wedge \alpha) \iff \alpha$

2. **idempotent law**  $(\alpha \vee \alpha) \iff \alpha$

3. **commutativity of  $\wedge$**   $(\alpha \wedge \beta) \iff (\beta \wedge \alpha)$

4. **commutativity of  $\vee$**   $(\alpha \vee \beta) \iff (\beta \vee \alpha)$

5. **associativity of  $\wedge$**   $((\alpha \wedge \beta) \wedge \gamma) \iff (\alpha \wedge (\beta \wedge \gamma))$

6. **associativity of  $\vee$**   $((\alpha \vee \beta) \vee \gamma) \iff (\alpha \vee (\beta \vee \gamma))$

7. **double-negation elimination**  $\neg(\neg\alpha) \iff \alpha$

8. **contraposition**  $(\alpha \supset \beta) \iff (\neg\beta \supset \neg\alpha)$

9.  $(\alpha \wedge \beta \supset \gamma) \iff (\alpha \supset \neg\beta \vee \gamma)$

10.  $(\alpha \supset \beta \vee \gamma) \iff (\alpha \wedge \neg\beta \supset \gamma)$

11. **de Morgan**  $\neg(\alpha \wedge \beta) \iff (\neg\alpha \vee \neg\beta)$

12. **de Morgan**  $\neg(\alpha \vee \beta) \iff (\neg\alpha \wedge \neg\beta)$

13. **distributivity of  $\wedge$  over  $\vee$**   $(\alpha \wedge (\beta \vee \gamma)) \iff ((\alpha \wedge \beta) \vee (\alpha \wedge \gamma))$

14. **distributivity of  $\vee$  over  $\wedge$**   $(\alpha \vee (\beta \wedge \gamma)) \iff ((\alpha \vee \beta) \wedge (\alpha \vee \gamma))$

15.  $\forall x (\neg\alpha) \iff \neg(\exists x \alpha)$

16.  $\exists x (\neg\alpha) \iff \neg(\forall x \alpha)$

17.  $\forall x (\alpha \wedge \beta) \iff (\forall x \alpha) \wedge (\forall x \beta)$

18.  $\exists x (\alpha \vee \beta) \iff (\exists x \alpha) \vee (\exists x \beta)$

19. if  $x$  is not free in  $\alpha$  then

$$\forall x (\alpha \vee \beta) \iff \alpha \vee (\forall x \beta)$$

$$\exists x (\alpha \wedge \beta) \iff \alpha \wedge (\exists x \beta)$$

$$\forall x \alpha \iff \alpha$$

$$\exists x \alpha \iff \alpha$$

20.  $\forall x \forall y \alpha \iff \forall y \forall x \alpha$

21.  $\exists x \exists y \alpha \iff \exists y \exists x \alpha$